Handout for Week 5

The Structure of Material Reason Relations

1. The first bit of structure is that **there *are* two kinds of reason relations**, and neither is in general definable in terms of the other. They are *implication* and *incompatibility*.
Q: Why? Why two, and not just one, or three? And if two, why just these two?

2. There is a fundamental *structural difference* between the two sorts of reason relations:

- Relations of *implication* must be substantially *non*symmetric—though they can include symmetric implication equivalences as special cases.
- Relations of *incompatibility* must be, in all cases, *symmetric*.

Q: Why? In particular, **why is *incompatibility* necessarily *symmetric***?
Why shouldn't commitment to *q* preclude entitlement to *p*, but commitment to *p* not preclude entitlement to *q*? Simonelli offers a cogent pragmatic argument. How should we understand it as related to the metaphysics of incompatibility, as appealed to in truthmaker semantics?

3. The tradition, including both Tarski and Gentzen, treat implication or consequence as a topological **closure** operator. Tarski uses:
<u>Kuratowski's Axioms for Topological Closure Operator (3 of 4):</u>
CO: $\Gamma \subseteq \mathrm{Con}(\Gamma)$.
MO: $\mathrm{Con}(\Gamma) \subseteq \mathrm{Con}(\Gamma \cup \Delta)$.
CT: $\mathrm{Con}(\mathrm{Con}(\Gamma)) = \mathrm{Con}(\Gamma)$.

<u>Gentzen-style</u>:
CO: $\Gamma, A |\!\sim A$

MO: $$\frac{\Gamma |\!\sim B}{\Gamma, A |\!\sim B}$$

CT: $$\frac{\Gamma |\!\sim A \quad \Gamma, A |\!\sim B}{\Gamma |\!\sim B}$$

4. Claim: Material (nonlogical) relations of implication do not generally satisfy Monotonicity (MO).

5. Failures of MO can generate failures of CT:
Here the presence of '(**not |**~)' where MO/CT requires '|~' shows failure of the principle.

Γ = Tweety is a bird.

A = Tweety flies.

Failure of MO:

B = Tweety is a penguin.

Tweety is a bird. |~ Tweety flies.
Tweety is a bird, Tweety is a penguin. (**not |**~) Tweety flies.

B'= Tweety is a nonpenguin.

Failure of CT:

Tweety is a bird |~ Tweety flies,   Tweety is a bird, Tweety flies |~ Tweety is a nonpenguin.
Tweety is a bird (**not |**~) Tweety is a nonpenguin.

6. Material (nonlogical) relations of implication do not generally satisfy Cautious Monotonicity (CM).

7. CT and CM are duals:

CM: $\dfrac{\Gamma|\sim A \quad \Gamma|\sim B}{\Gamma, A|\sim B}$

CT: $\dfrac{\Gamma|\sim A \quad \Gamma, A|\sim B}{\Gamma|\sim B}$

8. A *rational* sense of "implicit content":

When we express an implication Gentzen-wise, by writing "Γ|~A," we can think of it as indicating two aspects of the content of the premise-set Γ.
On the one hand, Γ is some set (usually finite) $\{G_1 \ldots G_n\}$ of sentences of the nonlogical language we are working in (so far).
Those sentences G_i, which are elements of the set Γ in the set-theoretic sense, can be thought of as expressing the *explicit content* of Γ. They are what the set Γ literally *contains*: its members.
Now the implication Γ|~A tells us that Γ implies A, so that in *another* sense A is part of the content of Γ. Γ *implies* A, and so "contains" it *implicitly*.
A is part of the *implicit content* of Γ in the *literal* sense of being *implied by* it.

In the pragmatic metavocabulary for reason relations offered last time, we read "Γ|~A" as saying that commitment to accept all of Γ precludes entitlement to reject A, and in that sense commitment to accept all of Γ *implicitly commits* one to *accept* A.

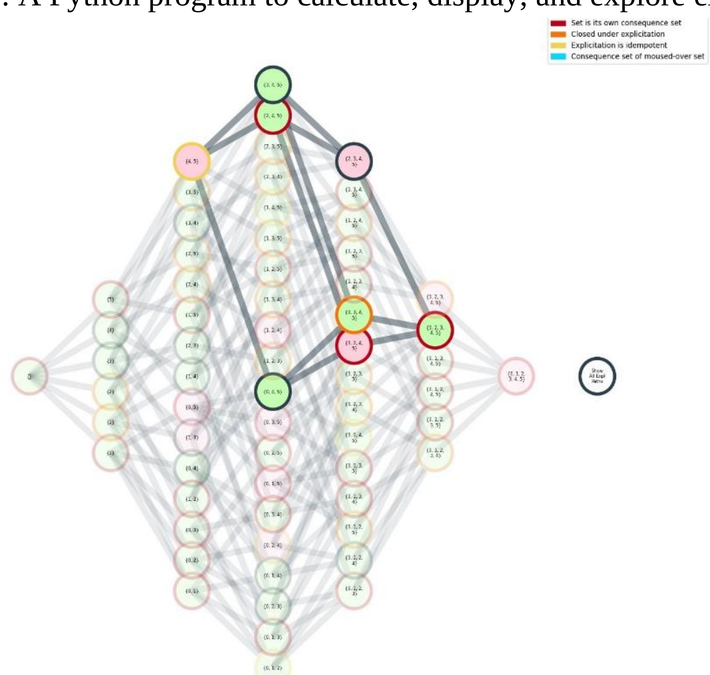
That is, commitment to accept Γ includes **implicit commitment to accept** (what we can now describe as) Γ's *rationally* **implicit content**.

9. ***Explicitation*** is moving a claimable (expressed by a sentence) from the right-hand side of the implication turnstile to the left-hand side.
When Γ|~A, we are interested in what is implied by Γ,A, compared to Γ.
We can think in these terms about the structural metainferential principles CM and CT as telling us something about the process of explicitation.
CM tells us that explicitation never *loses* consequences—that is, implicit content.
The premise-set that results from explicitation still has all the consequences, all the implicit content, that the original premise-set had.
CT tells us that explicitation never *adds* consequences—that is, implicit content.
The premise-set that results from explicitation *only* has the consequences, the implicit content, that the original premise-set had.

10. Together, CM and CT say that **explicitation is inconsequential**.
Making part of the *implicit* content of a premise-set *explicit* always yields a new premise-set with *exactly the same* implicit content (implications) as the original one.

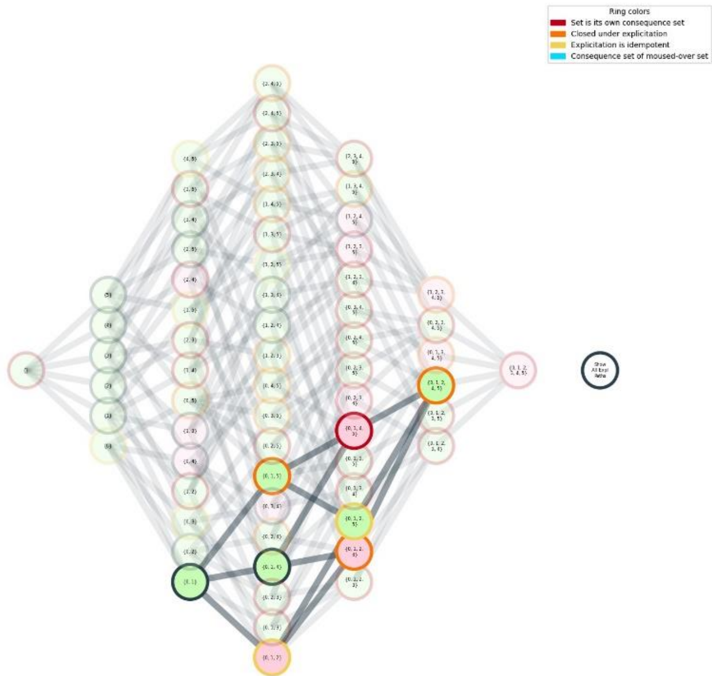
But in fact explicitation can make a significant difference to what is implied.
So we should reject at least the conjunction of CM and CT.

11. The Explicitator: A Python program to calculate, display, and explore explicitation paths.

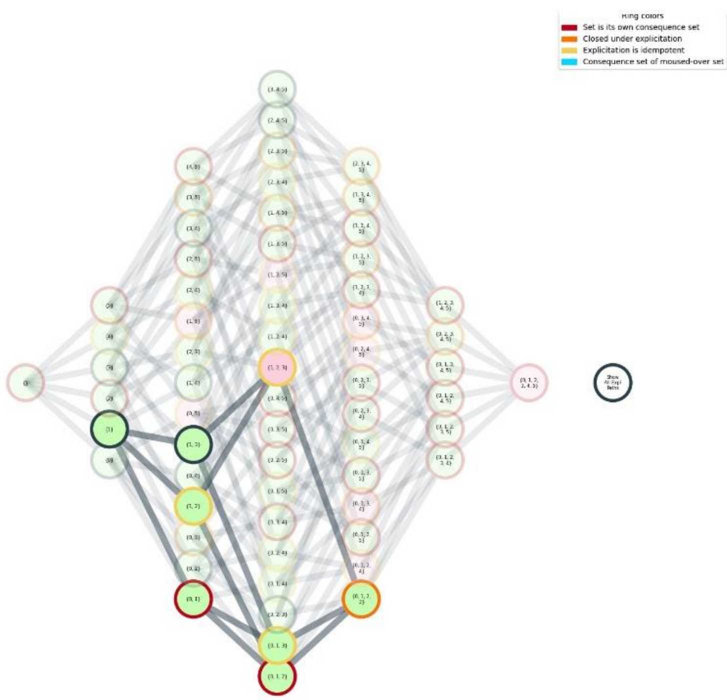


(4,5) is incoherent. Explicitating *any* of its consequences 0,2,3 cures the incoherence, however.

Yet if one explicitates *two* of them, specifically, 2 and either 0 or 3, the result is incoherent. But explicitating *all* of them (0, 2,3) restores harmony.



This example has the good property that the full initial explicitation is coherent, *and* there is a path to it that consists only of coherent consequence sets.



Here, we can calculate the number of paths that lead to the single incoherence, and compare that to the number of paths that stay coherent. We needn't be thinking about the probabilities of taking these paths being equal in order to be interested in the ratios of numbers of paths.